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THE EFFECT OF SIGHT MISALIGNMENT AND ANGLE OF ATTACK VARIATION.

S. H. Lechenbruch

ABSTRACT

In any toss bombing maneuver in which sight line and flight line prior to pull-up do not coincide, an impact error results. The amount of this error per mil of sight error is given by Equation (2) or Figure 2. The error may be corrected with the MPI adjustment dial by an amount determined from Equation (9) or Figure 3.

Sight misalignments depend on (a) the fixed orientation of the sight with respect to the airplane, and (b) the orientation of the airplane with respect to its flight line. The latter is the angle of attack, which is increased by a decrease in either dive angle or air speed, and therefore varies considerably since decreases in dive angle are usually accompanied by decreases in air speed. Differences in angle of attack may be determined from Equation (13) or from the nomogram of Figure 5.

If the effect of a sight misalignment is counteracted at some mean value of altitude, dive angle, and air speed, by an MPI adjustment $\Delta T_c / T_c$, the impact error at another altitude, dive angle, and air speed is the algebraic sum of that due to (a) the difference between the angles of attack at the mean value and second value of dive angle and air speed, (b) the sight error for which adjustment was made, and (c) the $\Delta T_c / T_c$ adjustment itself. Errors (b) and (c) are in opposite directions and partially offset each other, whereas (a) may affect the result considerably in either direction.

When the sight line is below the flight line, much greater bombing accuracy is achieved by a fixed readjustment of the sight itself than by an MPI adjustment. Under certain conditions it may be advantageous to overadjust the sight somewhat, bringing sight line above flight line, and correcting the resulting misalignment by a $\Delta T_c / T_c$ adjustment.

INTRODUCTION

In toss bombing, the flight path of the airplane before pull-up is assumed to be a collision course, i.e., a straight path directed toward the target. Since the flight path is determined in practice by means of a sight which is kept fixed on the target, the actual path can be a collision course only if the plane constantly flies along the sight line.

Sight misalignments may arise from the fact that for any aircraft the sight is usually adjusted to be correct for shooting of the machine guns. This adjustment is not suitable for bombs, since bombs leave the aircraft in a direc-

tion tangent to the flight path, while bullets, because of their high muzzle velocity, leave in the direction in which the guns are pointing. Differences between the sight and flight lines may also occur as a result of variations in the angle of attack, which for any type of plane depends on its weight, velocity, and dive angle. The errors resulting from such misalignments, and means of correcting them, form the subject of this paper.

Figure 1 represents the flight of plane and bomb for the case in which the line of flight makes a constant angle ϕ with the sight line, the sight being kept fixed on the target K' until pull-up. The vectors marked s represent instantaneous directions of the sight line, directed toward the target; those marked f, making an angle ϕ with the s-vectors, represent the corresponding instantaneous directions of flight. It will be noted that the path $\overline{AO}$ is slightly concave upward; it would be concave downward if ϕ had the opposite sense. For small angles ϕ, however, very little sacrifice of accuracy results from assuming that arc $\widehat{AO}$ is straight.

If A is the first altitude point, the values of dive angle (α), time-to-target (T_c), and airspeed (V), which are fed into the computer are determined by conditions along $\overline{AO}$, assuming that the dive angle gyro is aligned with respect to flight line. Barring other errors, therefore, the release time $T_P (= \overline{OP}/V)$ will be so determined as to cause a hit at H, on the extension of $\overline{AO}$, the corresponding slant range $\overline{OH}$ being $S = V \cdot T_c$. The horizontal impact error due to the misalignment is $\delta = \overline{K'H}$

The angle ϕ is considered positive when the sight line is above the flight line, and the error δ positive when it represents an impact beyond the target. In Figure 1, ϕ is positive and δ negative.

IMPACT ERROR RESULTING FROM SIGHT MISALIGNMENT

The exact relation between ϕ and δ is obtained at once by application of the sine law to triangle OHK':

$$-\delta = S \sin\phi / \sin(\alpha - \phi) = S / \sin\alpha (\cot\phi - \cot\alpha). \tag{1}$$

When $|\phi|$ is small compared with α, a close approximation is

$$\delta = -S\phi/\sin\alpha, \text{ or } -\delta/\phi = S \csc\alpha, \tag{2}$$

ϕ being in radians. The amount by which (2) is in error as compared to the exact expression (1), is given approximately by the ratio ϕ/α; thus for ϕ values up to 1° or 17 mils, (2) is accurate to within 5% for α as small as 20°, and to within 10% for α = 10°.

Figure 2 is a contour map, in polar co-ordinates, of the relation between S and α corresponding to several fixed values of the ratio $-\delta_\phi/\phi$, δ_ϕ being the impact error caused by sight error ϕ. The corresponding rectangular coordinates of points on these curves are the horizontal range $(S\cos\alpha)$ and second altitude $(R_2 = S\sin\alpha)$: hence the curves may be considered pictorially as the loci of spatial release points corresponding to given $-\delta_\phi/\phi$. the

target being at the origin. The value of $-\delta_\phi/\phi$ for a given release point may be obtained by interpolation between consecutive curves, so that Figure 2 is in one sense a graph of $-\delta_\phi/\phi$ as a function of release point. (Several other contour maps of this type are included in this paper.

On converting (2) to rectangular coordinates, it becomes apparent that the curves of Figure 2 are arcs of circles which pass through the origin, with centers on the vertical axis and radii $-\frac{1}{2}\delta_\phi/\phi$. ϕ being in radians, or $-500\,\delta_\phi/\phi_{mils}$. It will be noted that δ_ϕ/ϕ is independent of air speed and pull-up acceleration.

According to the exact equation (1), δ_ϕ is actually proportional, not to ϕ, but to $1/(\cot\phi - \cot\alpha)$, which differs significantly from ϕ in radians when α and $|\phi|$ are of comparable magnitude, i.e., for very small α or very large ϕ. The graph of Figure 2 therefore becomes accurate for <u>all</u> values of α and ϕ if the "feet per mil" values with which the curves are labeled are considered as values, not of $-\delta_\phi/\phi_{mils}$ but of $-1000\,\delta_\phi\cdot(\cot\phi - \cot\alpha)$.

<u>CORRECTION FOR SIGHT MISALIGNMENT BY MPI ADJUSTMENT</u> $\Delta T_c/T_c$

The error due to non-coincidence of sight and flight lines would be eliminated, causing a hit on the target K' (Fig. 1), if the T_c value fed into the computer were adjusted to correspond to a target at H' instead of H. This would mean a relative increase $\epsilon_c = \Delta T_c/T_c$ (or a relative decrease $-\Delta T_c/T_c$), where $V\Delta T_c = \overline{HH'}$, and $\Delta T_c/T_c = \overline{HH'}/\overline{OH}$.

Application of the sine law to the triangle $HH'K'$, on the usual assumption that the parabolic arc $K'H'$ has negligible curvature, yields at once

$$V\Delta T_c = -\delta\sin(\alpha+\theta_g')/\sin\theta_g' = -\delta\,\Theta\sin\alpha, \tag{3}$$

i.e.,
$$\epsilon_c = \frac{V\Delta T_c}{S} = -\frac{\delta\,\Theta\sin\alpha}{S},$$
or
$$\delta/\epsilon_c = -S/\Theta\sin\alpha, \text{*} \tag{4}$$

where θ_g' is the angle between trajectory and collision course at their intersection, and

$$\Theta = \frac{\sin(\alpha+\theta_g')}{\sin\alpha\sin\theta_g'} = \cot\theta_g' + \cot\alpha \tag{5}$$

Formula (4) or its equivalent has been encountered in every problem involving correction of an impact error. The value of Θ in terms of S, V, α, and K (where Kg is pull-up acceleration) is implied in Report OD-SP-105, Equa-

* The negative sign in (4) holds only when ϵ_c represents the adjustment required to <u>offset</u> the error δ. When it represents that required to <u>produce</u> the error δ, the sign of the right member of (4) is <u>positive</u>.

tions (9), (3), and (6):

$$\Theta = \left[\mu/K + \sqrt{\mu/K}\,\sqrt{1+2\beta} + \csc^2\alpha\,(-1+\sqrt{1+2\beta})\right](1+\sqrt{1+2\beta})\tan\alpha/2\beta \tag{6a}$$

$$= \frac{\sqrt{\mu/K}\,\tan\alpha}{-1+\sqrt{1+2\beta}}\left(\sqrt{\mu/K} + \sqrt{1+2\beta}\right) + \sec\alpha\csc\alpha, \tag{6b}$$

where $\mu = K - \cos\alpha$, and

$$\beta = \frac{Sg}{V^2}\cdot\frac{\mu}{K}\sin\alpha \tag{7}$$

Substitution of (1) into (4) gives the correction required to effect a given ϕ:

$$\epsilon_c = \frac{V\Delta T_c}{S} = +\frac{\Theta}{\cot\phi - \cot\alpha} = \frac{\cot\theta_f' + \cot\alpha}{\cot\phi - \cot\alpha} \tag{8}$$

Again, when $|\phi|$ is small compared with α, a close approximation is

$$\epsilon_c \approx \Theta\,\phi, \quad\text{or}\quad \frac{\epsilon_c}{\phi} = \Theta \tag{9}$$

The accuracy of (9) relative to (6) is similar to that given in the statement following equation (2).

A formula equivalent to (8) was already derived in Report OD-SP-40 in the course of a general discussion of angles.

Equation (9) may be solved for S by solving (6b) for $\sqrt{1+2\beta}$ and using the relations $\beta = \frac{1}{2}\left[(\sqrt{1+2\beta})^2 - 1\right]$, $S = \beta\cdot V^2K/\mu g\sin\alpha$

The result is

$$S = \frac{V^2}{g\sin\alpha}\;\frac{K}{\mu}\;\frac{N}{D}\left(1+\frac{1}{2}\,\frac{N}{D}\right) \tag{10}$$

where

$$N = \tan\alpha\,\sqrt{\mu/K}\,(1+\sqrt{\mu/K}),\quad D = \epsilon_c/\phi - \sqrt{\mu/K}\,\tan\alpha - \sec\alpha\csc\alpha \tag{11}$$

Figure 8 is a contour map, based on (10), of ϵ_c/ϕ, as percent-change in T_c per mil of sight error, as a function of release point. K is held constant at the value 8, the variation of ϵ_c/ϕ with K being quite small. The range and altitude scales involve a factor η which is unity when $V = 350$* knots. In accordance with (10), which implies that S varies as V^2 for fixed ϵ_c/ϕ and α, the graph may be used for any air speed V by substituting $\eta = (V/350\text{ knots})^2$. At 250 and 500 knots, $\eta \approx 1/2$ and 2, respectively.

* actually $250\sqrt{2} \approx 353.6$ knots.

As in the case of Figure 2, Figure 3 as such becomes inaccurate for very small α or very large ϕ, but becomes accurate for all α and ϕ if the "% per mil" figures given are considered as values, not of ϵ_c/ϕ mils, but of $1000\,\epsilon_c(\cot\phi - \cot\alpha)$.

According to (9), Figure 3 may also be interpreted as a contour map of the much-used function Θ. When used as such, the "% per mil" figures must be reconverted into "units per radian" by multiplication by .01/.001 = 10. The curve marked 0.8% per mil, for example, is as well the locus of Θ = 8.

Figure 4 is a contour map, based on (4), of $|\delta_c/\epsilon_c|$ as a function of release point, δ_c being the impact error corresponding to the change ϵ_c. From Figure 4 may be read the amount of displacement of the impact point per percent change in T_c, both changes having the <u>same algebraic sign</u>. Furthermore, the reciprocals of the $|\delta_c/\epsilon_c|$ figures on the curves represent the percent change in T_c required to offset each foot of impact error; and when so interpreted, δ_c and ϵ_c have <u>opposite algebraic signs</u>. The graph is of a general nature, applicable not only to the sight error problem but to all problems involving correction of an impact error by adjustment of T_c. The air speed factor η = (V/350 knots)^2 is included in the range, altitude, and contour scales.

By comparing equations (2), (4), and (9), and noting that $(\epsilon_c/\phi)(\delta/\epsilon_c) = \delta/\phi$, it is seen that for any given release point the value of δ_ϕ/ϕ, as given by Figure 2, is the product of the values of ϵ_c/ϕ, as in Figure 3, and of δ_c/ϵ_c as in Figure 4. The curves of Figure 4 have been constructed, not by solving (4) for δ directly — this would have yielded a set of cubic equations — but rather from Figures 2 and 3, by determining points of intersection of curves in Figure 3 with circles of appropriate radii similar to those in Figure 2.

<u>VARIATION OF ANGLE OF ATTACK</u>

Application of the preceding section, and of Figures 2 and 3, requires that the value of the sight error ϕ for any specific case, or at least the difference between ϕ values for two specific cases, be known. The effective sight error, however, depends not only on the fixed orientation of the sight with respect to the airplane, but also on the angle of attack, or orientation of the plane with respect to its flight line. The variation of the latter with air speed and dive angle is appreciable and must be taken into account.

The angle of attack ϕ_a is given by the formula

$$\phi_a = \frac{C\,W\cos\alpha}{V^2} - k. \tag{12}$$

where W is the gross weight of the plane, and C and k are constants for any one plane. The value of k also depends on the choice of a reference line in the airplane, and for purposes of calculating <u>differences</u> in angles of attack

one may set $k = 0$ with no loss of generality:

$$\phi_a = \frac{c\, W \cos \alpha}{V^2} \tag{13}$$

It is found generally that C has the same value for all planes of a series, as TBM-1 and TBM-1C.

The nomogram of Figure 5 gives this relative angle of attack ϕ_a, as in (13), as a function of C, W, V and α. The calibrations on lines I, II, IV, and VI, have been determined from values of log C, log W, log V, and log $\cos \alpha$, respectively. Points on line III have been pre-determined for each plane at its nominal weight *, thus obviating the use of lines I and II whenever the gross weight is approximately nominal.

Lines III through VII have been so spaced that V and α may be entered in either order. This property is illustrated by the example appearing on the nomogram. The point on line V marked "F6F, 350 knots" may be joined with several points in turn on line VI to obtain ϕ_a on line VII for different dive angles at the fixed air speed; or the point marked "F6F, 40°" may be joined with points on line IV to obtain ϕ_a for different air speeds at the fixed dive angle. The relative convenience of the two methods depends on whether one is concerned more with variation of ϕ_a with dive angle or with air speed.

According to (13), the angle of attack, while independent of range, is increased by a decrease in either air speed or dive angle. Hence the range of variation in ϕ_a for any given plane is widened by the fact that decreases in dive angle are in practice usually accompanied by decreases in air speed.

APPLICATION OF THEORY

The preceding sections suggest two alternative methods of correcting for sight misalignment at a given range, dive angle, and air speed: (a) direct adjustment of the sight until flight and sight lines coincide; or (b) integrator adjustment, i.e., changing T_a by a percentage ϵ_a determined from Figure 3.

The adjustment required, however, depends in either case on the values of α and V, and in case (b) on S as well. Since it is not feasible to make frequent readjustments of either type in any one plane, in practice a fixed adjustment must be made, such as would completely offset the sight error at some chosen intermediate or modal value of S, α, and V. At other values an impact error will generally result, the magnitude of which depends on the method used -- a fixed sight adjustment, a fixed integrator adjustment, or some combination of the two. The most efficient method is of course the one which minimizes this impact dispersion.

The following example illustrates the results of a fixed integrator adjustment: The sight installed in an F6F plane at nominal weight (12,400 lbs.) is misaligned by -20 mils at slant range $\bar{S} = 7500$ feet, dive angle $\bar{\alpha} = 40°$, and air speed $\bar{V} = 350$ knots. A fixed percent-adjustment in T is made in an amount

* As obtained from OSRD Reports 2264 (CIT/UWC 3), 2271-2275 (CIT/UWC 4-8), 2382 (CIT/UWC 25), and 2347 (CIT/UWC 26).

ust sufficient to offset the -20 mil sight error at ($\bar{S}$, $\bar{\alpha}$, $\bar{V}$), causing a hit. It is desired to determine the resulting impact error at slant range S_1 = 12,000 feet, dive angle $\alpha_1 = 60°$, and air speed V_1 = 375 knots.

From Figure 3 the required percent-adjustment in T_c per mil at 7000 feet, 40°, and 350 knots (Point $\bar{P}$) is $\epsilon_c/\phi = 0.75\%$ per mil, so that for $\phi = -20$ mils the fixed adjustment is $\epsilon_c = -15.0\%$. In other words, under these conditions a 15% decrease in T_c, by itself, would displace the impact point by an amount equal and opposite to the displacement which would result from a -20 mil sight error alone.

At another range, dive angle, and air speed these two displacements, while oppositely directed, will generally be unequal in magnitude; and in addition, the angle of attack will generally be different. Thus the impact error at (S_1, α_1, V_1) will be the algebraic sum of three displacement components: (a) the difference $\Delta\phi_a$ between the angles of attack at ($\bar{\alpha}$, $\bar{V}$) and (α_1, V_1) is a change in sight error and therefore causes an impact displacement δ_a; (b) the original sight error $\phi = -20$ mils, by itself, would result in a positive (beyond-target) impact error δ_ϕ; (c) the adjustment $\epsilon_c = -15\%$, by itself, would give a negative (short-of-target) error δ_c.

(a) The difference in angle of attack is determined from Figure 5. For an F6F plane at nominal weight, this nomogram gives ϕ_a = 14.9 mils at $\bar{\alpha}$ = 40°, $\bar{V}$ = 350 knots, and 8.5 * mils at α_1 = 60°, V_1 = 375 knots. Hence $\Delta\phi_a$ = -6.4 mils. But from Figure 2, at α_1 = 60° and S_1 = 12,000 feet (Point Q), $-\delta/\phi$ = about 14 feet per mil, so that the impact displacement due to change in angle of attack is δ_a = +90 feet.

(b) From this same ratio $-\delta/\phi$ = 14 feet per mil, it follows that the -20 mil sight error in itself causes an impact error δ_ϕ = +280 feet.

(c) Finally, at 12,000 feet, 60°, and 375 knots, the value of δ_c/ϵ_c may be read from Figure 4. As there defined, $n = (375/350)^2 = 1.148$, so that at 375 knots, 12,000 feet = $\frac{12,000}{1.148}$ x n = 10,450 n feet. The point q determined by this slant range and 60 deg. dive angle lies between the curves marked 10n and 15n, slightly nearer the latter, so that approximately δ_c/ϵ_c = 13n = 14.9 ft. per %. The -15.0% adjustment in T_c therefore displaces the impact point by about δ_c = -220 feet.

The resultant impact error at (S_1, α_1, V_1) is then

$$\delta_a + \delta_\phi + \delta_c = +90 + 280 - 220 = +150 \text{ feet;} \qquad (14)$$

i.e., the bomb may be expected to fall 150 feet beyond the target under the given conditions.

The effect of angle of attack variation may be seen to be of considerable importance. If δ_a were neglected in this example, (b) and (c) alone would yield an impact error of +60 feet, or only 40% of that given by (14). Quite often even the algebraic sign of the resultant error is changed by this effect.

* Determined by extending line VII and using lower extension scale.

According to the wording of the hypothesis, this example may at first appear to typify only the integrator adjustment method. But if the original misalignment were given not as -20 but as -30 mils, of which 10 mils were corrected by a sight adjustment and the balance by an integrator adjustment, the solution would be identical, yet the problem would appear more general, involving a combination of sight and integrator adjustments. ¶ The above example illustrates the method of determination of the extent to which a fixed adjustment fails to offset a sight misalignment at ranges, dive angles, and air speeds other than the central value for which the adjustment is made. The mode and extent of variation of this residual impact error with range, dive angle, and air speed is illustrated in Figures 6, 7 and 8, for both the sight and integrator adjustment methods and various combinations of the two. In these graphs are plotted the results of fixed adjustments so determined as to eliminate completely the effect of the sight error at the model values $\bar{S} = 7500$ feet, $\bar{\alpha} = 40°$, $\bar{V} = 350$ knots, as in the example above. They are based on Figures 2 - 5 and on the formulas underlying them. Explanation of the derivation, interpretation, and use of Figures 6, 7, and 8 follows:

At $\alpha = \bar{\alpha}$ and $V = \bar{V}$, let ϕ be that portion of the original sight error which remains after any direct sight adjustment is made; ϕ is then the error to be offset by an integrator adjustment. If this latter adjustment consists of a change in T_c in the ratio ϵ_c, then by (9),

$$\epsilon_c = \phi \cdot \bar{\Theta}, \tag{15}$$

where $\bar{\Theta}$ is the value of Θ when $S = \bar{S}$, $\alpha = \bar{\alpha}$, and $V = \bar{V}$. As determined in the example, $\bar{\Theta} = 0.75\%$ per mil.

At any other value of S, α, and V, the error δ on the ground consists of three components analogous to those lettered (a), (b), (c), in the example above: (a) By (2) and (13), the impact displacement due to change $\Delta\phi_a$ in angle of attack is

$$\delta_a = -S\,\Delta\phi_a \csc\alpha = -\frac{SCW}{\sin\alpha}\left(\frac{\cos\alpha}{V^2} - \frac{\cos\bar{\alpha}}{\bar{V}^2}\right) \tag{16a}$$

where C, W, $\bar{\alpha}$ and $\bar{V}$ are known constants. (b) Again by (2), that due to the model sight error ϕ alone is

$$\delta_\phi = -S \cdot \phi \csc\alpha \tag{16b}$$

(c) Finally, by (4) and (15), that due to the integrator adjustment ϵ_c alone is

$$\delta_c = +S\epsilon_c / \Theta \sin\alpha = +S\,\phi \cdot \csc\alpha\; \Theta/\bar{\Theta} \tag{16c}$$

The resultant error δ is the sum of (16a), (16b), and (16c):

$$\delta = \delta_a + \delta_\phi + \delta_c = S\csc\alpha\left[-\Delta\phi_a + \phi(-1 + \Theta/\bar{\Theta})\right] \tag{17}$$

where $\Delta\phi_a$ is a function of α and V, and $\bar{\Theta}$ is a constant.

When the entire correction is made through a sight adjustment, then $\phi = 0$ and hence $\delta = \delta_a$ as given by (16a). Figure 6 is a contour map of δ_a as a function of release point, for air speeds 350 ± 25 knots, for an F6F plane (at nominal weight) whose sight is adjusted for $\bar{\alpha} = 40°$ and $\bar{V} = 350$ knots. It will be noted that the curves for any one air speed approach parallelism in the direction corresponding to the dive angle α for which $\delta_a = 0$ at that air speed — i.e., for which $\cos\alpha / V^2 = \cos 40° / (350 \text{ knots})^2$. While constructed on the basis of F6F data, the errors given by Figure 6 are accurate to within 5% for the SB2C, and to within 15% for the TBM and F4U. The actual value of δ_a for any type of plane is obtainable from Figure 6 and the relation

$$\delta_a = \delta_{a(F6F)} \quad cW/c_{F6F} W_{F6F},$$

which follows from (16a).

When, on the other hand, part or all of the correction is made through an integrator adjustment (ϵ_c), δ is the sum of this δ_a, as plotted in Figure 6, and an error $\Delta\delta = \delta_\phi + \delta_c = S\phi\csc\alpha(-1 + \Theta/\bar{\Theta})$, proportional to ϕ, which would result if angle of attack were constant. The amount of this second error per mil, i.e., $\Delta\delta/\phi$ is plotted pictorially in Figure 7 as a function of release point, again for air speeds 350 ± 25 knots. The 350-knot curve marked $\Delta\delta/\phi = 0$ is of course identical with that curve of the family of Figure 3 which passes through the point $\bar{S}$ = 7500 feet, $\bar{\alpha} = 40°$, $\bar{V}$ = 350 knots, and is therefore the locus of $\epsilon_c/\phi = \bar{\Theta}$ = 0.75% per mil.

The residual error δ for the general case is given by the sum of the errors as given in Figures 6 and 7, the latter being a function of the sight misalignment ϕ to which the integrator adjustment is applied. When the sight itself is adjustable, ϕ may be given any pre-determined value by means of a preliminary sight adjustment. Figure 8 is a set of contours of δ, obtained by combining Figures 6 and 7, for different values of ϕ between -20 and +20 mils. The scale is the same as in Figures 6 and 7, but ranges and dive angles outside the most probable operating limits * are excluded in order to facilitate analysis of the errors likely to be encountered in practice. The central graph (c), which is for $\phi = 0$, is identical with Figure 6, except that the contour interval used in Figure 8 is 50 feet.

The complete solution of the above example is obtainable from Figures 6, 7, and 8. The point Q in Figures 6, 7, and 8(a) represents (S_1 = 12,000 feet, $\alpha_1 = 60°$). Interpolation between consecutive error curves for V_1 = 375 knots, with ϕ = -20 mils, gives respectively $\delta_a \approx +90$ feet, $\Delta\delta \equiv \delta_\phi + \delta_c \approx$ (-3 feet per mil) × (-20 mils) = +60 feet, $\delta \approx +150$ feet, in exact agreement with the solution (14).

A study of Figure 8 sheds much light on the relative merits of the sight adjustment and integrator adjustment methods and of various combinations thereof. The general trend of Figure 8 may be predicted from Figures 6 and 7.

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* The operating limits assumed here are as follows: Minimum dive angle 20°, minimum slant range, 4000 feet; maximum dive angle 60°, maximum second altitude 10,000 feet, and maximum slant range that which, according to Report OD-SP-105 (Fig.2) would correspond to 100-foot horizontal error at 350 knots for Mod 0 bomb director. The point $\bar{P}$ ($\bar{S}$ = 7500 ft., $\bar{\alpha} = 40°$) is close to the "center of gravity" of this operating region.

When either α or V increases, according to Figures 6 and 7, δ_a increases, whereas $\Delta\delta^*$ decreases or increases according as ϕ is positive or negative; these two error components therefore partially offset each other, as regards variation with α and V, when ϕ is positive, but increase the total error when ϕ is negative. This observation is borne out quantitatively by Figure 8. At $\phi = -20$ mils (Figure 8a) the curves are densely packed, indicating wide variation of residual error; within the operating limits and for ± 25-knot variation in V, δ is seen to vary approximately between -200 and +150 feet. As ϕ increases the curves become sparser, until at $\phi = +20$ (Figure 8e) δ becomes practically independent of air speed. At $\phi = +10$ mils (Figure 8d), while being somewhat more dependent on air speed, δ is limited to +50 feet throughout almost the entire operating region and at a fixed slant range seems to be almost independent of α. The corresponding variation at $\phi = 0$ (Figure 8c) is also between the limits of ± 50 feet, and more independent of slant range. In all three curves (c), (d) and (e), the worst error conditions occur in the vicinity of 20° and maximum range, the bomb falling short by an amount which approaches 100 feet. These results would, of course, be altered somewhat if different values of $\bar{S}$, $\bar{\alpha}$, and $\bar{V}$ were used in the adjustment, but the general trend would be entirely similar.

CONCLUSIONS

Aircraft whose sights are adjusted to be correct for machine gun use have sight misalignments which average about -30 mils for toss bombing. In view of Figure 8, correction of errors of such magnitude by means of an MPI ($\Delta T_c / T_c$) adjustment alone is highly inadvisable, as this method makes the impact point very sensitive to changes in range, dive angle, and air speed. Adjustment of the sight itself brings much more bombing accuracy within wide limits, and seems to give an adjustment roughly independent of slant range.

Equally favorable results, however, may be achieved if a combination of sight and integrator adjustments is used. In this method the sight is over-adjusted until misaligned by about +10 to +20 mils, and this misalignment is in turn corrected by an MPI adjustment. The optimum sight adjustment depends on the choice of a model value of $\bar{S}$, $\bar{\alpha}$, and $\bar{V}$, but when these are 7500 feet, 40°, and 350 knots, the minimum variation of impact error with dive angle is reached at an overadjustment of approximately +10 mils; with air speed at approximately +20 mils; and with slant range at approximately 0 mils (sight adjustment alone).

The effect of this combined adjustment is to offset the effect of changes in angle of attack by means of an integrator adjustment so determined as to counteract such variation. An additional advantage of this method lies in the fact that, in practice, steeper dives are generally accompanied by greater air speeds. Since an increase in either dive angle or air speed increases the error due to changes in angle of attack, it follows that a sight adjustment alone would make the impact point doubly sensitive to changes in α or V. The combination method reduces the variation of δ with both α and V. However,

* Except for small dive angles.

if the minimization of the variation of δ with slant range at fixed values of α and V is considered more important, only the sight adjustment, i.e., $\phi = 0$, should be made.

The above choice of modal values $\bar{S}$, $\bar{\alpha}$, $\bar{V}$, about which the adjustments are to be made, appears to be satisfactorily centralized with respect to usual operational limits. Any decrease in $\bar{S}$ below 7500 feet would make positive values of $\Delta\delta/\phi$ (Figure 7) considerably more probable than negative values; and a slight increase, perhaps to 8500 feet, would yield a somewhat better balanced distribution of errors.

In practice, however, greater accuracy in the sight adjustment is achieved at short ranges. Since angle of attack is independent of range, adjustment of the sight at $\bar{\alpha}$, $\bar{V}$, and a range less than $\bar{S}$ would not affect the result. The MPI adjustment ($\Delta T_c / T_c$) could then be made at $\bar{S}$, $\bar{\alpha}$, and $\bar{V}$.

It is also recommended that this combination method of sight correction be tested in the field, to determine the agreement of theory and practice at different values of ϕ. In any such tests, however, one caution is believed to be in order. The impact errors with which we are here concerned are comparable in magnitude with errors due to such factors as pilot aiming and the use of approximate ψ-functions. A difference between two observed impact errors, for the same plane under different conditions, will be approximately free of components due to other factors, and is therefore much more reliable for this purpose than any single absolute observed error.

The results of this analysis, furthermore, de-emphasize the necessity for exact alignment of the flight and sight lines. An approximate alignment will generally suffice provided the error is on the side of over-adjustment, which is subsequently corrected through an MPI adjustment.

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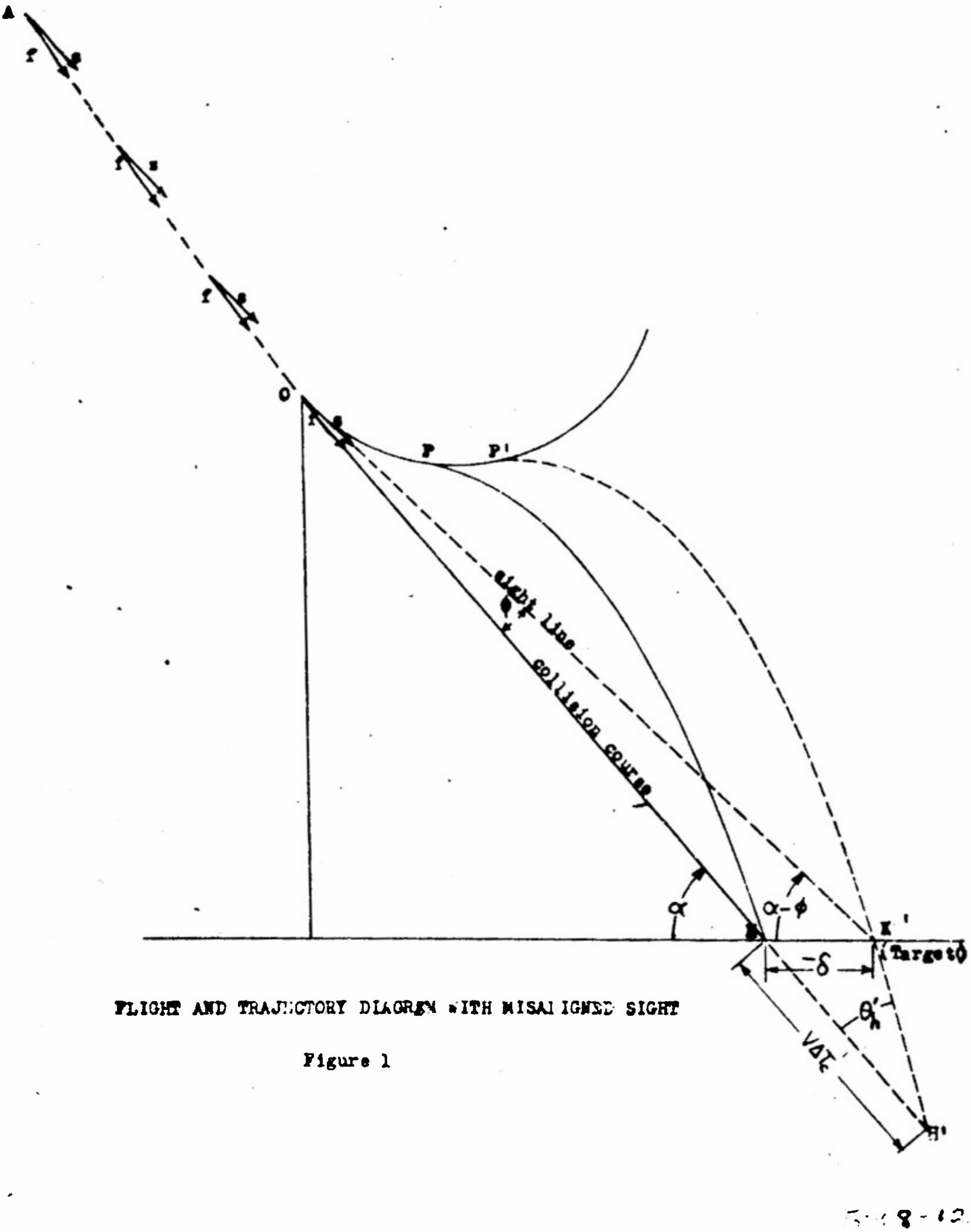

FLIGHT AND TRAJECTORY DIAGRAM WITH MISALIGNED SIGHT

Figure 1

Spatial Contour Map of:
HORIZONTAL IMPACT ERROR (δ_ϕ) RESULTING FROM A GIVEN SIGHT ERROR (ϕ)

(Range-dive angle contours of constant ratio δ_ϕ/ϕ, as indicated)

Independent of air speed (V) and of pull-up acceleration (K)

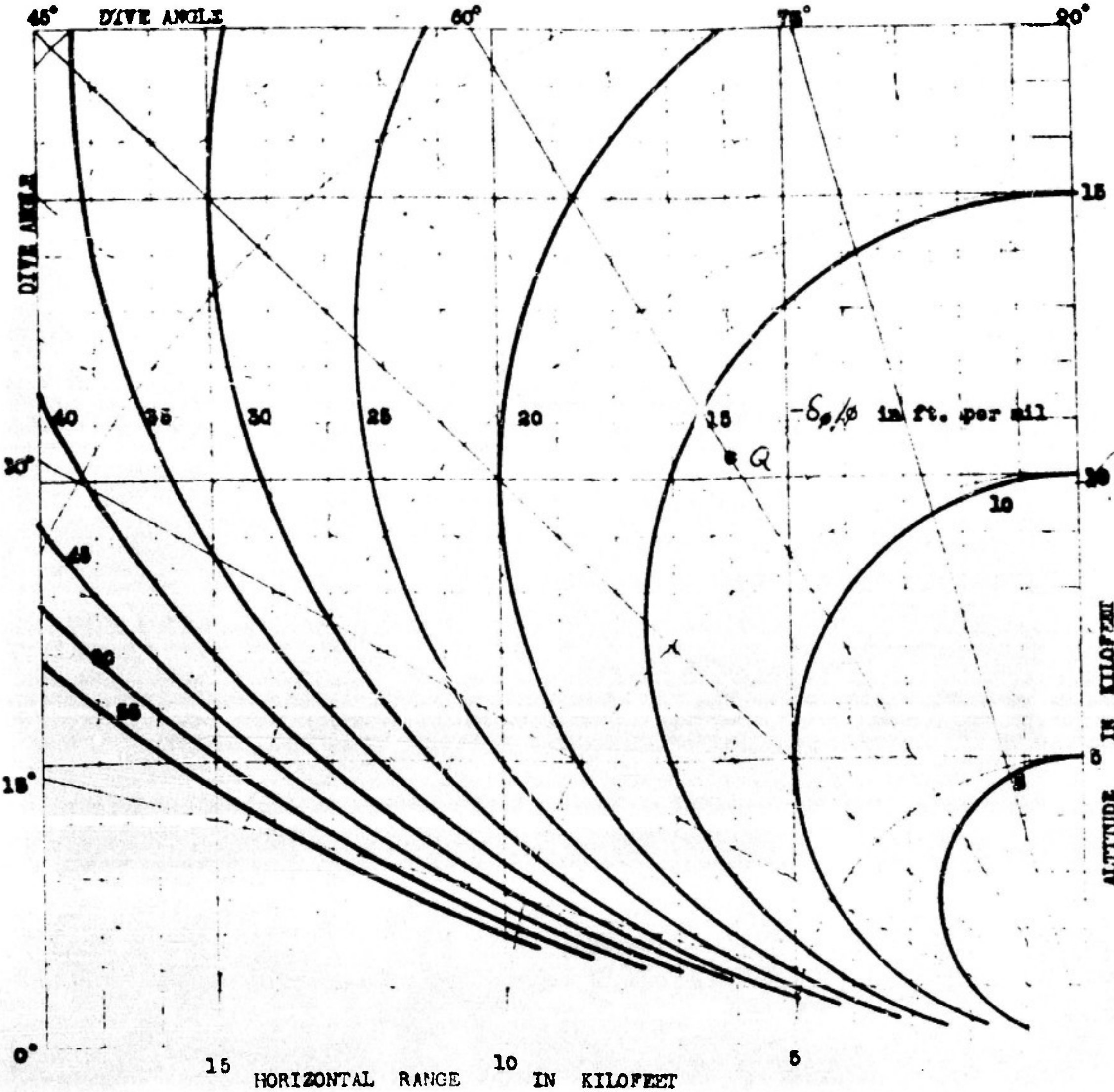

Figure 2

Spatial Contour Map of:

MPI ADJUSTMENT (ϵ), I.E., PERCENT CHANGE IN TIME TO TARGET, REQUIRED TO OFFSET GIVEN SIGHT ERROR (ϕ)

V air speed	$n=(V/350\text{knots})^2$
[illegible] knots	[illegible]
350 knots	1
500 knots	2

(Range-dive angle contours of constant ratio ϵ/ϕ)

N=3 (Slight dependency on K)

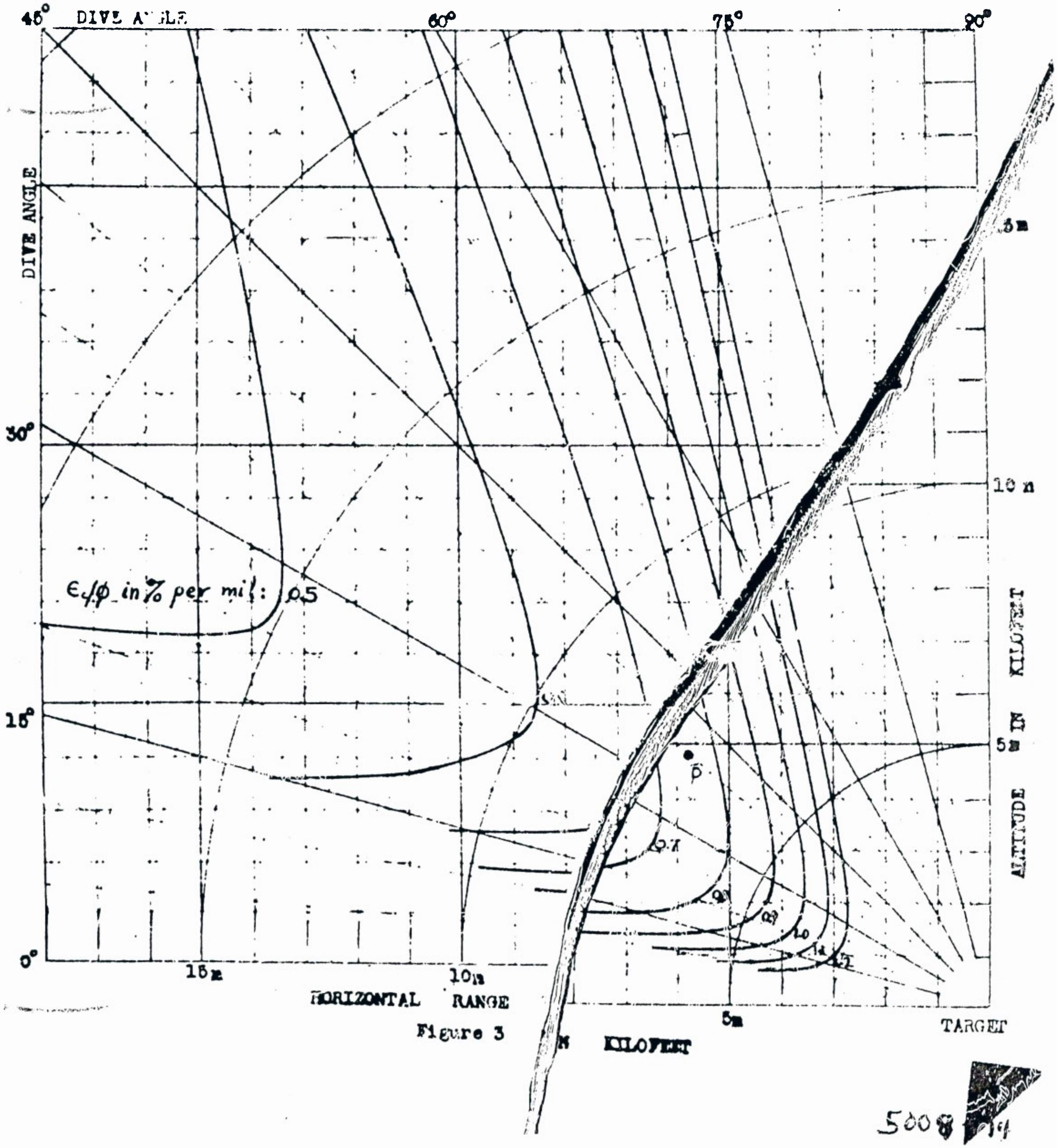

Figure 3

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Spatial Contour Map of
HORIZONTAL IMPACT ERROR (δ_c) RESULTING FROM GIVEN PERCENT CHANGE (ϵ) IN
TIME TO TARGET

(Range-dive angle contours of constant ratio δ_c/ϵ_c)

K=3 (Slight dependency on K)

V=air speed	a=(V/350knots)^2
250 knots	½
350 knots	1
500 knots	2

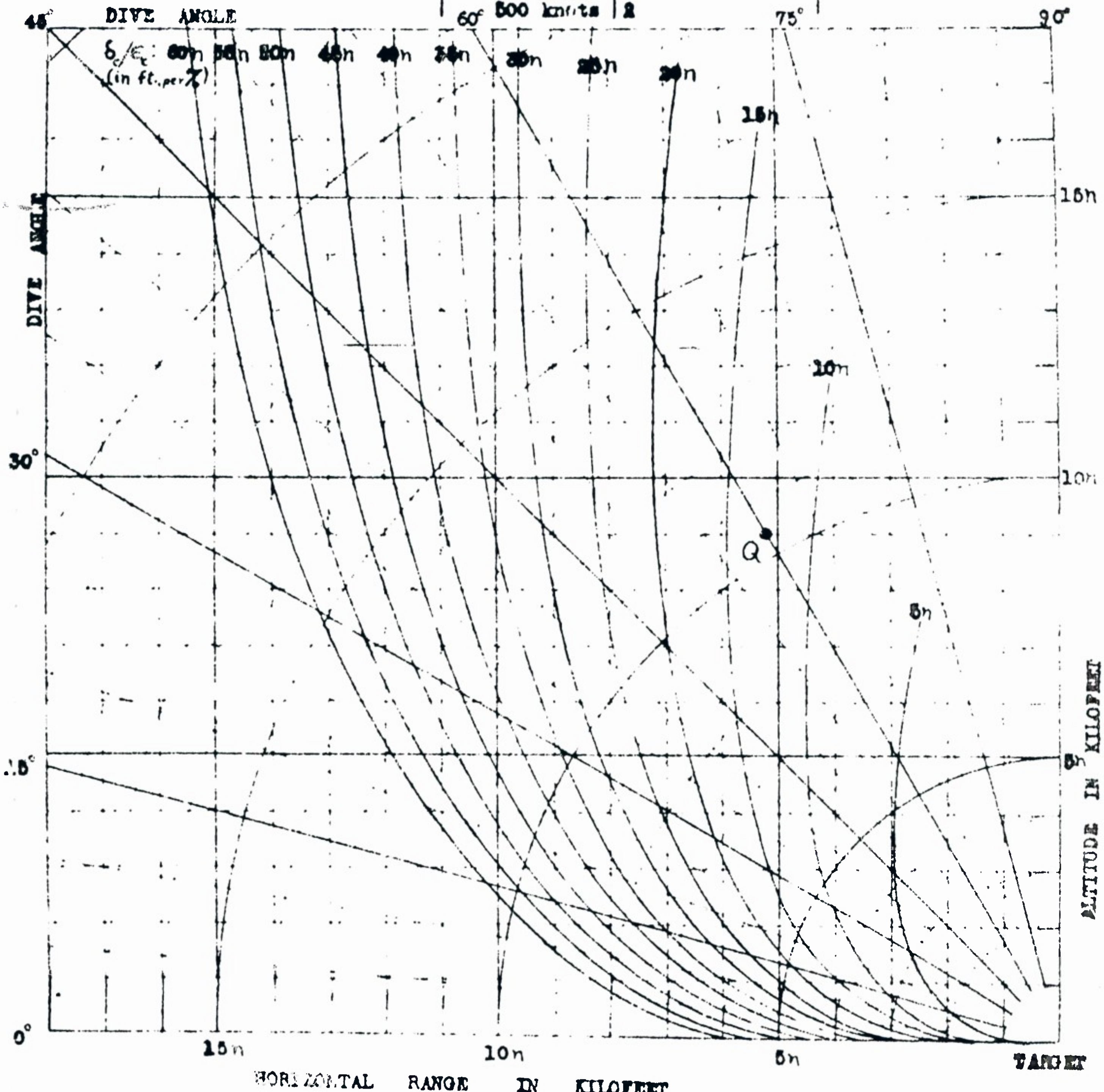

Figure 4

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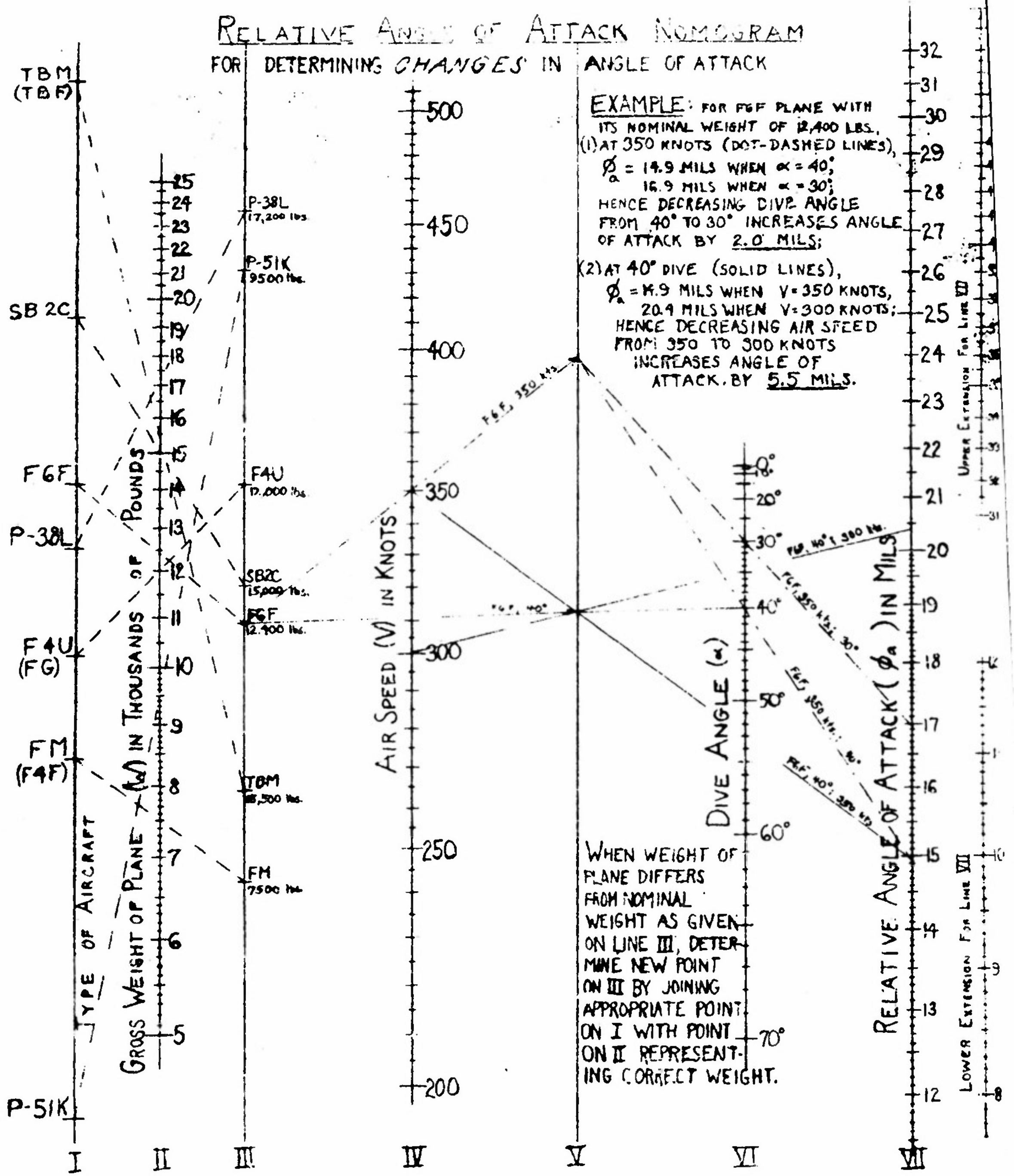

RELATIVE ANGLE OF ATTACK NOMOGRAM
FOR DETERMINING CHANGES IN ANGLE OF ATTACK
EXAMPLE: FOR F6F PLANE WITH ITS NOMINAL WEIGHT OF 12,400 LBS.,
(1) AT 350 KNOTS (DOT-DASHED LINES),
ϕa = 14.9 MILS WHEN α = 40°;
16.9 MILS WHEN α = 30°;
HENCE DECREASING DIVE ANGLE FROM 40° TO 30° INCREASES ANGLE OF ATTACK BY 2.0 MILS;
(2) AT 40° DIVE (SOLID LINES),
ϕa = 14.9 MILS WHEN V = 350 KNOTS,
20.4 MILS WHEN V = 300 KNOTS;
HENCE DECREASING AIR SPEED FROM 350 TO 300 KNOTS INCREASES ANGLE OF ATTACK BY 5.5 MILS.
WHEN WEIGHT OF PLANE DIFFERS FROM NOMINAL WEIGHT AS GIVEN ON LINE III, DETERMINE NEW POINT ON III BY JOINING APPROPRIATE POINT ON I WITH POINT ON II REPRESENTING CORRECT WEIGHT.
TYPE OF AIRCRAFT
TBM (TBF)
SB 2C
F6F
P-38L
F4U (FG)
FM (F4F)
P-51K
GROSS WEIGHT OF PLANE (W) IN THOUSANDS OF POUNDS
P-38L 17,200 lbs.
P-51K 9500 lbs.
F4U 12,000 lbs.
SB2C 15,000 lbs.
F6F 12,400 lbs.
TBM 15,500 lbs.
FM 7500 lbs.
AIR SPEED (V) IN KNOTS
DIVE ANGLE (α)
RELATIVE ANGLE OF ATTACK (ϕa) IN MILS
UPPER EXTENSION FOR LINE VII
LOWER EXTENSION FOR LINE VII
I
II
III
IV
V
VI
VII

Figure 5

Spatial Contour Map of:
HORIZONTAL IMPACT ERROR (δ_a) RESULTING FROM CHANGES IN ANGLE OF ATTACK,
WHEN SIGHT IS PROPERLY ALLIGNED AT DIVE ANGLE 40° AND AIR SPEED 350 KNOTS.

(Range-dive angle contours of constant δ_a, as indicated)

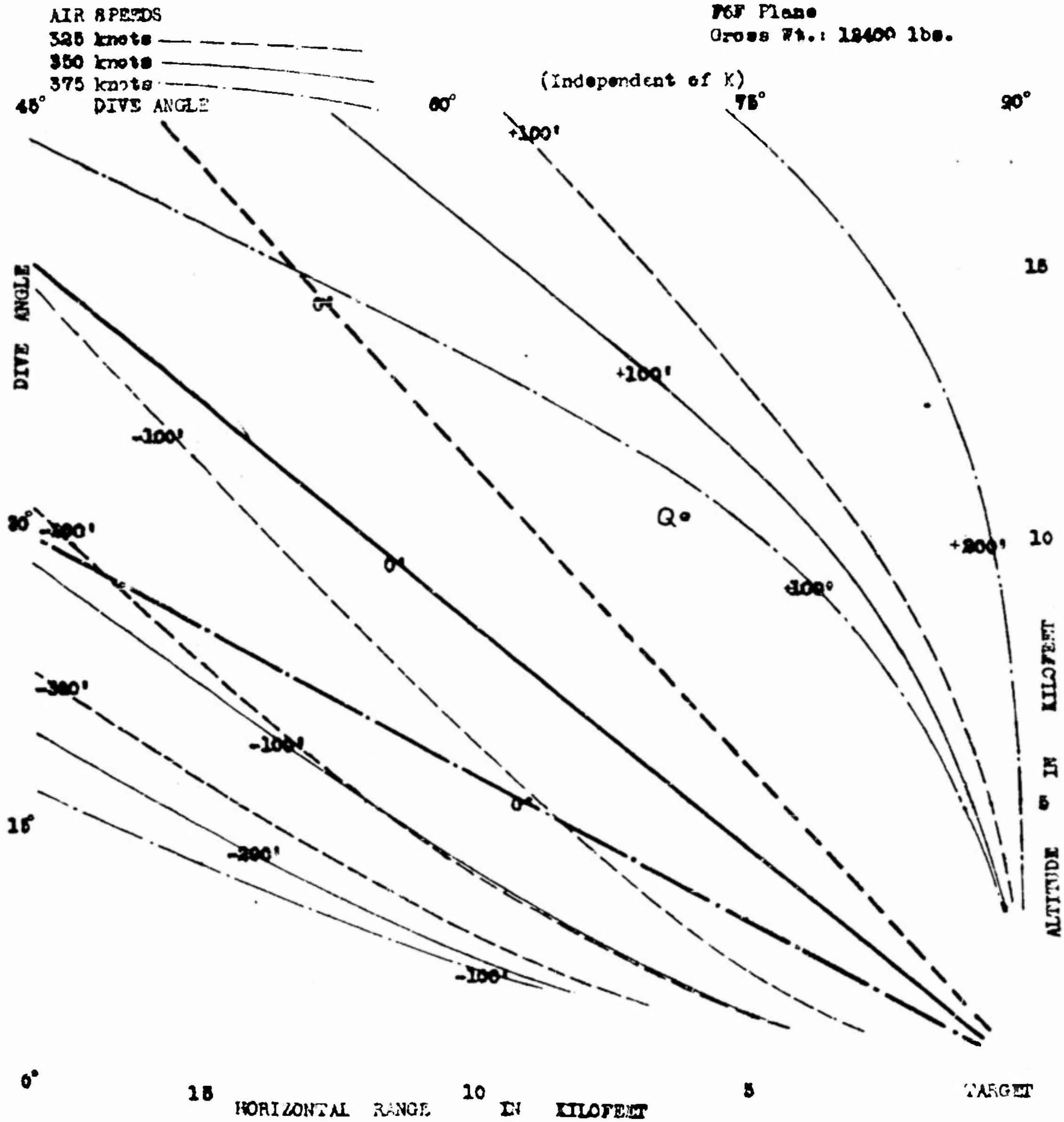

Figure 6

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Spatial Contour Map of
HORIZONTAL IMPACT ERROR ($\Delta\delta = \delta_\phi + \delta_c$), NOT INCLUDING THAT DUE TO ANGLE OF ATTACK, RESULTING FROM FIXED MPI ADJUSTMENT (ϵ_c) SO DETERMINED AS TO OFFSET A SIGHT ERROR (ϕ) AT SLANT RANGE 7.5 KILOFEET, DIVE ANGLE 40°, AND AIRSPEED 350 KNOTS

(Range-dive angle contours of constant $\Delta\delta/\phi$, as indicated)

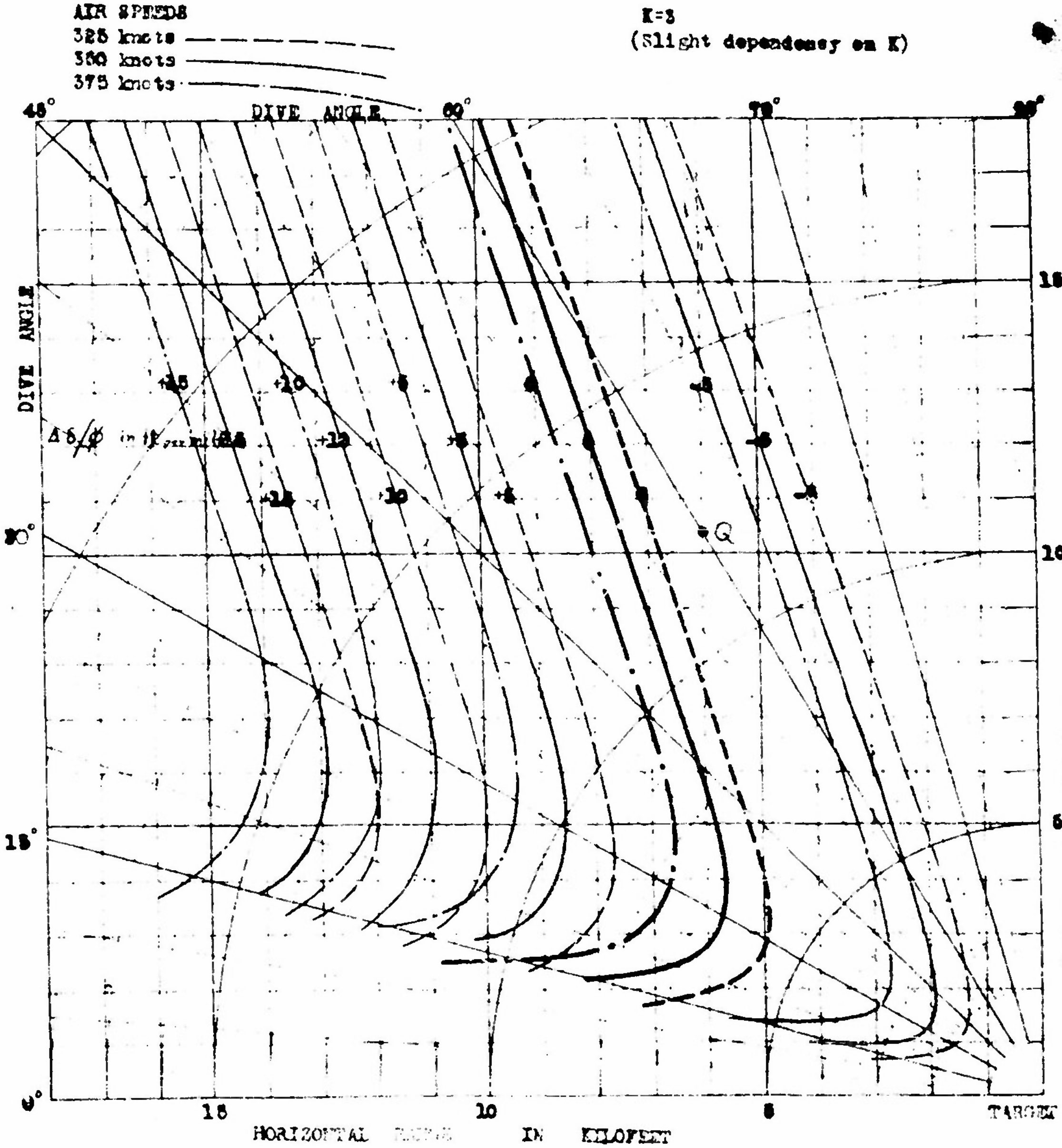


Figure 7

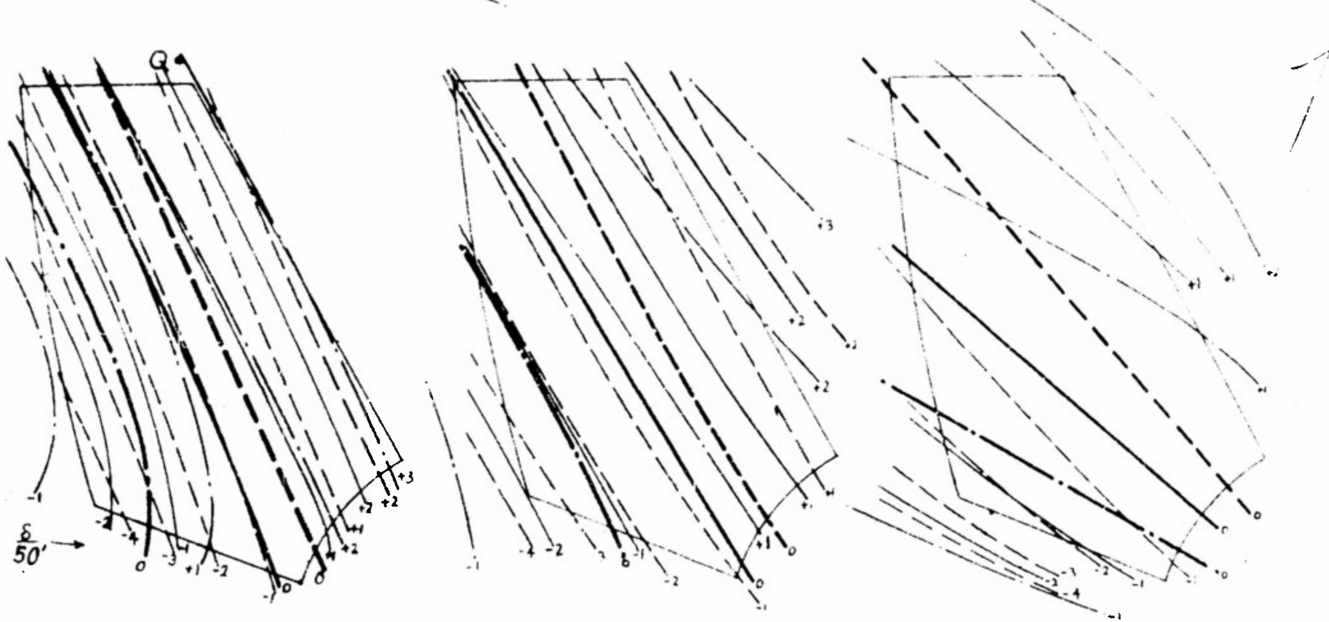
5008-18

Spatial [illegible]ontour [illegible] (composite of Figures 6 [illegible] 7) of:
HORIZONTAL [illegible] ($\delta = \delta_2 + \Delta\delta$) RESULTING FROM
FIXED [illegible] ADJUSTMENT SO DETERMINED AS TO OFFSET A
SIGHT [illegible] (ϕ) AT SLANT [illegible] 75 KILO[illegible],
DIVE [illegible] 40°, AND AIR SPEED 350 KNOTS
([illegible]ange-dive angle contours of constant δ, within operational limits,
given in units of 50')

AIR SPEEDS
525 knots – – – – –
350 knots ———
375 knots – · – · –

F6F Plane
Gross Wt. 12,400 lbs.

K=3



(a) $\phi = -20$ mils

(b) $\phi = -10$ mils

(c) $\phi = 0$

Figure 8

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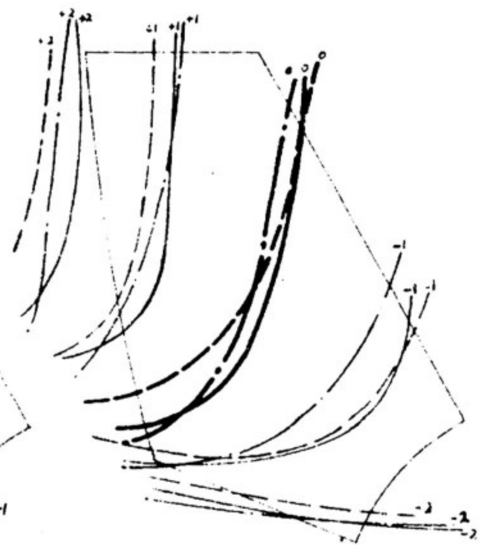
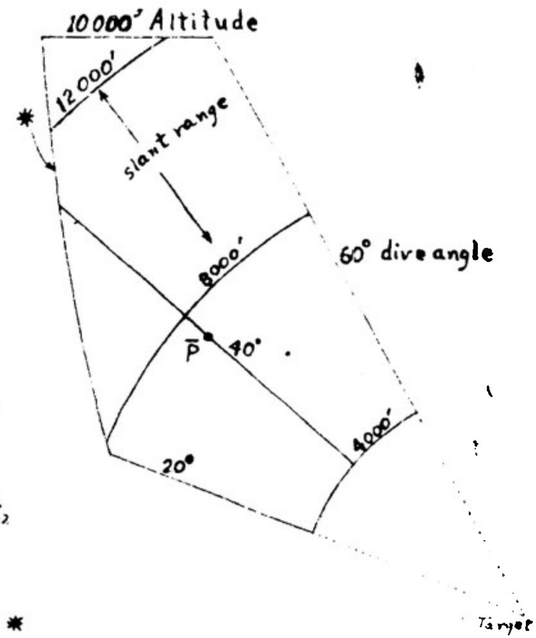
(d) $\phi = +10$ mils



(e) $\phi = +20$ mils

Key



* Theoretical maximum range for Mod. 0 bomb director at 350 knots

THE END